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While the accretion power in astrophysics has been studied in many astronomical environments, the "spin–down power" is often neglected. In this essay I demonstrate that the spin–down power alone may drive a rotating system from sub-critical condition to critical condition with a small but finite probability. In the case of an isolated spinning-down neutron star, the star may undergo a quark–hadron phase transition in its center and become observable as a soft gamma repeater or a cosmological gamma–ray burst. For a spinning–down white dwarf, its Chandrasekhar mass limit will decrease and may reach the stellar mass, then the star explodes to a type Ia supernova. Gravitational wave detectors may be able to test these models.

Accretion power plays a fundamental role in astrophysics [1], and is believed to be the energy source for active galactic nuclei, Galactic X-ray sources, novae and supernovae. What about spin–down power? Most stars are spinning down while losing angular momentum. The decrease in centrifugal force leads to an increase in the stellar central density $\rho_c$. $\Delta\rho_c$ is generally small unless a star spins down from an initial frequency close to its Keplerian frequency $\Omega_K$ (the frequency at which the star begins to shed matter near its equator). The spin–down power is often neglected for two reasons. First, most stars spin much slower than their $\Omega_K$. However, as I shall show in this essay, if a star has a critical central density or a critical mass, the spin–down power alone can drive the star to have a catastrophic transition from sub-critical condition to critical condition, and cause a release of large amount of energy in observable forms. Second, the conventional way of treating rotating stars misses the evolution of an individual star. Stars of one type are often treated as a set, and relations such as $M_* - \rho_c$ are usually solved in the literature. Rotation leads to a "mass increase" in the $M_* - \rho_c$ plot, which is actually the mass difference between two stars with the same $\rho_c$ at different angular velocities. These theoretical results are difficult to test observationally. The evolution of an individual rotating star, in terms of tracing $\rho_c$ and overall structural changes over time, has rarely been studied. That is likely why we are missing some interesting explanations of many astrophysical phenomena.

To solve the structure of a rotating relativistic star, Hartle developed [2,3] a perturbation solution based on the Schwarzschild metric of a static, spherically symmetric object. Rotation distorts the star away from spherical symmetry. By treating $\rho_c$ as an input parameter and $\Delta M_*$ as a perturbation, $M_* - \rho_c$ relations and the "mass increase" due to rotation can be solved for different rotational frequencies $\Omega$ (as seen by a distant observer) [3–5]. We have noted that, instead of deriving $M_* - \rho_c$ relations for a family of neutron stars, tracing the evolution of an individual star gives better insight [6]. Along with an increase in $\rho_c$, the overall structure, chemical composition and spin–down behavior of a star are modified. In principle, we can derive a set of equations parallel to those of Hartle, with $M_*$ as an input parameter and $\Delta\rho_c$ as the perturbation, and solve $\rho_c$ and stellar structures at different $\Omega$. In practice, it is more convenient to exploit Hartle's method to solve $\Delta\rho_c$ for an individual star, using the approximation that a star has a constant gravitational mass. A clever way to do this is to first plot $M_* - \rho_c$ relations at different $\Omega$ using Hartle's method [3–5], then cross these curves

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with a line of constant mass. The projected $\rho_c$'s are those of an individual star at different $\Omega$s. The structural change of a spinning–down star can thus be solved self-consistently when the Equations of State (EOS) of the star are known.

The EOS of quark matter is much softer than that of neutron matter because of the QCD asymptotic freedom. A neutron star containing a quark core is thus more compact and has a larger $\Omega_K$ than a normal neutron star. If the initial spin period of a neutron star is $P_i \approx 20$ ms, e.g. that of the Crab pulsar, it will have a central density increase $\Delta\rho_c/\rho_c \sim 0.1\%$ in its life time. Assuming a critical density $\rho_{\rm cr}$ for a phase transition, only those neutron stars born with $\rho_{\rm cr}(1-\Delta\rho_c/\rho_c) < \rho_c < \rho_{\rm cr}$ would have the chance to evolve from sub-$\rho_{\rm cr}$ to $\rho_{\rm cr}$. If $\rho_{\rm cr}$ lies between the lower and upper limits for the central density, i.e. $\rho_{\rm l} < \rho_{\rm cr} < \rho_{\rm u}$, a small but finite fraction of neutron stars will undergo the transition at a rate [7]

$$R = \frac{\rho_{\rm cr}(\Delta\rho_c/\rho_c)}{\rho_{\rm u} - \rho_{\rm l}}\, R_{\rm NS} \simeq 10^{-5} \left(\frac{P_i}{20\,{\rm ms}}\right)^{-2} \left(\frac{R_{\rm NS}}{10^{-2}}\right) {\rm yr}^{-1}{\rm galaxy}^{-1}, \tag{1}$$

where $R_{\rm NS}$ is the neutron star birth rate. Clearly these events can be observed on a regular basis with millions of galaxies in our view.

A catastrophic phase transition inside a spinning–down neutron star can happen on a time scale of seconds. In such a short time, the star collapses from a radius of $r \sim 15$ km to $r \sim 10$ km, associated with a sudden spin–up. The gravitational energy released $E \simeq 10^{53}\Delta r/r$ ergs is large enough to power a cosmological gamma-ray burst (GRB) [7]. Observed GRBs have total energy of $10^{51} - 10^{54}$ ergs (assuming the energy is emitted isotropically), which were explained with colliding neutron stars [8]. The event rate is estimated to be $10^{-5}$ yr$^{-1}$ galaxy$^{-1}$ if the emission is isotropic, and can be much higher if the emission is highly beamed. Equation (1) can still account for the GRBs even if they are beamed, as long as faster initial spins are sought. The faster initial spin of a neutron star naturally offers a large angular momentum, that helps beam the electromagnetic radiations.

Quark–hadron phase transition is different from water–vapor phase transition even though it is likely to be first order. It has an additional freedom of whether a proton deconfines to *uud* quarks or a neutron becomes *udd* quarks. Hence, electric charges are not conserved in each of the two phases, although overall charge neutrality is achieved through leptons. Consequently, the two phases are not necessarily separated by gravity [9]. If this were the case, the phase transition will happen slowly on a time scale of $10^5$ years, and we should be able to observe $\sim$1 event in our Galaxy at any moment. Gravitational energy is released at an average rate of $10^{40}$ ergs s$^{-1}$ during this slow phase transition. Most of this energy is released via neutrino emission, and only a tiny fraction is used to heat the star up to a surface temperature of $3\times10^6$ K, yielding a soft X-ray luminosity of $\sim 10^{35}$ ergs s$^{-1}$. This is still 25 times more luminous than the sun [10]! Unlike many Galactic X-ray sources powered by accretion in binary systems, these types of X-ray sources can be isolated objects.

While the fluid core of the star is contracting, stress builds up in the solid crust. The cracking of the crust releases bursts of energy that can be observed as Soft Gamma Repeaters (SGRs) [11]. SGRs are X-ray transient sources associated with young ($10^4$ yr) supernova remnants (SNRs). They are also usually quiescent X-ray emitters (with $kT \sim 1$ keV, $L_{\rm X} \sim 10^{35}$ ergs s$^{-1}$). So far four SGRs have been discovered in the Galaxy, and one in the Large Magellanic Cloud. Two of these SGRs have characteristic ages $\tau_c = \Omega/2|\dot\Omega|$ ($\sim 10^3$ yrs) $<$ SNR age ($\sim 10^4$ yrs) [12]. We already know that $\tau_c$ should be an upper limit for a pulsar's age. How does it reconcile with the age of an SNR, which is responsible for the birth of the pulsar in the first place? This can be explained easily with the picture of phase transition. The phase transition tends to spin–up a neutron star while making it more compact and easier to brake, and hence $\tau_c$ may *underestimate* the true age while it is an *upper* limit for normal neutron stars.

Now let us take a look at the role of spin–down power in the progenitors of type Ia supernovae (SNe Ia). SNe Ia are believed to be explosions of Chandrasekhar mass carbon–oxygen White Dwarfs

(WDs) in binary systems [13]. However, detailed observations have ruled out almost any accretion rate in binary evolutions [14,15], leading SN Ia theory to a paradox [16]. Instead of accretion, the spin–down power can drive the central density of a WD to the critical density for carbon ignition and trigger an explosion. The key point is that a rotating WD has a larger Chandrasekhar mass limit $M_{\rm Ch}{\simeq}M_{\rm Ch,0}(1+3T/|W|)$ [17], where $M_{\rm Ch,0}$ is for non-rotating WDs; $T$ and $W$ are rotational and gravitational energy, respectively.

Observed rotation of WDs (most of which have masses $\sim 0.6M_\odot$) is generally small with $T/|W|{\lesssim}10^{-5}$ [17]. Although there are not enough data to give a mass–rotation relation for WDs, it is likely that more massive WDs have higher $T/|W|$ ratios for two reasons. First, they have smaller radii; second, they have suffered less mass loss and thus less angular momentum loss. We will assume that the progenitor of a WD loses its outer layers and leaves behind a rotating core (a pre-WD), which has a uniform density before collapsing into a WD. A $1.4M_\odot$ pre-WD has a radius 1.326 times that of a $0.6M_\odot$ pre-WD, and its moment of inertia $I{\simeq}0.4Mr^2$ is 3 times larger. After the WDs are formed, more massive ones have smaller radii $r_{1.4M_\odot}{\sim}0.3r_{0.6M_\odot}$ [17]. It is easy to see that for the collapsed WDs $T_{1.4M_\odot}{\sim}80T_{0.6M_\odot}$ and $W_{1.4M_\odot}{=}0.25W_{0.6M_\odot}$. Hence, $3T/|W|{\sim}10^{-2}$ for a $1.4M_\odot$ WD. As a result, $M_{\rm Ch}{=}1.01M_{\rm Ch,0}$ and is about $1.414M_\odot$ if $M_{\rm Ch,0}{=}1.400M_\odot$.

The observed mass distribution of about 200 WDs [18,19] can be roughly fit with a power law mass function $N(M){\sim}M^{-3}$ with $M$ between $0.6M_\odot$ and $1.3M_\odot$. Although no WDs more massive than $1.3M_\odot$ are observed in these samples, it is likely that the high mass tail of the distribution from $1.400M_\odot$ to $1.414M_\odot$, extrapolated from the observed mass function, constitutes $\sim$0.1% of the total WD population. The spin-down time scale of these WDs is $\sim 10^9$ yr [17], during which $M_{\rm Ch}$ evolves from $1.414M_\odot$ to $1.400M_\odot$. As a result, about 0.1% of all WDs will have the chance to evolve from sub-Chandrasekhar mass to their mass limit and undergo catastrophic events solely due to spin–down. The number of WDs in a galaxy is $\sim 10^{10}$, among which $10^7$ will be in the high mass tail with $M_*{>}M_{\rm Ch,0}$ and will undergo catastrophic events due to spin–down within $10^9$ yr. Hence, the event rate is $10^{-2}$ yr$^{-1}$ galaxy$^{-1}$. The fate of these WDs is bifurcated. If their central densities reach the critical density for carbon ignition, they will become SNe Ia. If not, they will tend to collapse into black holes or neutron stars. The latter may be the origin of isolated millisecond pulsars.

In binary evolution, in addition to the mass accretion, a WD can be spun–up. A WD can grow significantly more massive than $M_{\rm Ch,0}$ without exploding, if the transfer of angular momentum is efficient. It is possible that a WD explodes long after the accretion ceases. It is intuitive to define an "effective accretion rate" $\dot{M}_{\rm eff}{=}3TM_{\rm Ch,0}/(\tau|W|)$, where $\tau$ is the time scale. In the case of accretion and spin–up, $\dot{M}_{\rm eff}$ is negative and is associated with a positive real accretion rate. In the case of spin–down without accretion, $\dot{M}_{\rm eff}$ is positive and it describes how fast $M_{\rm Ch}$ approaches the gravitational mass of the WD. If a WD is spun–up to a high rotation with $T/|W|{\sim}0.1$, then the gravitational radiation rather than the viscosity dominates the dissipation process, and $\tau{\sim}10^3{-}10^7$ yr rather than $10^9$ yr [17]. Correspondingly, $\dot{M}_{\rm eff}{\sim}10^{-4}{-}10^{-8}M_\odot$ yr$^{-1}$, which is comparable to real mass accretion rate.

Analysis of the non-spherical shapes of nova remnants suggest that WDs can have such fast rotations [20]. Hence, the transport of angular momentum during the accretion process can be very efficient, which can be easily understood by realizing that part of the huge amount of *orbital* angular momentum of the binary system is converted to *spin* angular momentum of the WD during accretion.

Traditional mass accretion models for SNe Ia predict generally weak gravitational waves (GWs), because the exploding WD is slowly rotating and the explosion is nearly spherical. For the model described in this paper, it is possible that the progenitors of some SNe Ia are rapidly rotating WDs, which undergo asymmetric explosions and produce strong GWs. The collapsing or exploding rapidly rotating compact stars can produce GWs as strong as those from stellar mergers, but the wave forms are very different [21]. Hence, it is possible to test different models for GRBs and SNe Ia in future GW observations such as the LIGO experiment. However, it should be noted that GWs are emitted more or less isotropically in a GRB, while the electromagnetic radiation is likely to be highly beamed (with a beaming factor 100–1000) in the angular momentum direction. LIGO may need to accumulate 100–1000 events before seeing a gravitational wave signal at the same time as a GRB.

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